More About *At Least*

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**Goal of this talk:**  
Comparing *at least* to another scalar particle, *even*

**Outline:**  
1. Summary of CLA talk (epistemic vs. concessive *at least*)  
2. Some background on *even*  
3. Similarities and differences between *at least* and *even*  
4. Cross-linguistic evidence for the relatedness of *at least* and *even*  
5. Towards an explanation of the differences  
6. A tentative proposal  
7. Conclusion and outlook

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**Epistemic vs. Concessive *At Least***

**Epistemic at least**  
(Krifka 1999, Geurts and Nouwen 2007, Büring 2008)

1. Mary wrote *at least* four novels.  
   - *The speaker is uncertain about exactly how many novels Mary wrote*  
2. Mary won *at least* a silver medal.

**Concessive at least**

3. Mary didn’t win a gold medal, but *at least* she won a silver medal.  
   - *Although winning a silver medal is less preferable than winning a gold medal, a silver medal is satisfactory*
**Epistemic at least**

(Based on Krifka 1999, Geurts and Nouwen 2007, Büring 2008)

**Truth conditions**

\[ \exists q \in C[q \geq p \land q(w) = 1] \]

“there is a proposition q which ranks higher than or as high as the target proposition p and which is true”

**Conventional implicature**

\[ \exists w'[\text{Epist}(w,w') \land \exists q \in C[q > p \land q(w') = 1]] \]

“it is epistemically possible that some proposition q that ranks higher than p is true”

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**Concessive at least**

**Truth conditions**

\[ p(w) = 1 \]

“The target proposition p is true”

**Conventional implicatures**

i. \[ \forall r, r' \in C[r > r' \leftrightarrow r \text{ is preferred to } r'] \]

“The scalar ranking reflects a preference ranking”

ii. \[ \exists q \in C[q > p] \]

“There is a proposition q that ranks higher than p”

iii. \[ \exists q \in C[q < p] \]

“There is a proposition q that ranks lower than p”

\[ \rightarrow p \text{ is better than some other alternatives but not the best (“settle for less”)} \]

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**Even and Polarity**

- “Positive” contexts

(4) She has even read *Syntactic Structures*.
   – top of the scale (hardest / least likely)

- “Negative” contexts

(5) She hasn’t even read *Syntactic Structures*.
   – bottom of scale (easiest / most likely)

- Ambiguity in some cases:

(6) I doubt that she has even read *Syntactic Structures*. 

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2. Some Background on *Even*
Two Theories of *Even*

  - *even*
  - interaction with polarity is due to scope
  - in “negative” contexts, *even* can take scope over DE operator (which causes scale reversal)

  - *two evens*: “regular” *even* and NPI *even*
  - “regular” *even* is associated with top of scale
  - NPI *even* is associated with bottom of scale
  - NPI *even* has to be in the scope of DE operator

Lexical Differentiation

- Many languages use different lexical items

- Dutch
  - regular *even* = *zelfs*
  - NPI *even* = *zelfs maar*

(7) Hij heeft *zelfs* alle vragen goed beantwoord.
   ‘He even answered all questions correctly.’

(8) Ik denk niet dat hij *zelfs maar* een vraag goed beantwoord heeft.
   ‘I don’t think he even answered one question correctly.’

3. Similarities and Differences between *At Least* and NPI *Even*

**Synonymy?**

- *At least* often seems to be interchangeable with NPI *even*:
  
  (9) Did I answer *at least / even* one question correctly?

  (10) If you answer *at least / even* one question correctly, you’ll pass.

- Both *at least* and NPI *even* are associated with the bottom end of a scale.

- But there are some (sometimes subtle) differences.
Polarity

• **NPI even is an NPI** (well, duh!)

  (11) He didn’t answer even one question.
  (12) * He answered even one question.

• **At least is a PPI**

  (13) He answered at least one question.
  (14) * He didn’t answer at least one question.
  but OK
  - if *at least one question* takes wide scope
  - as ‘echo’ negation (Baker 1970, Seuren 1976)

Biased Questions

Positive bias:
(15) Did I answer at least one question correctly?

Negative bias:
(16) Did I answer even one question correctly?

• **Cf. Some vs. Any** (Lakoff 1969)

Positive bias:
(17) Did I answer some questions correctly?

Negative bias:
(18) Did I answer any questions correctly?

Conditionals: Promises vs. Threats

Threat:
(19) If you make even one mistake, I’ll break your legs.
(20) # If you make at least one mistake, I’ll break your legs.

Promise:
(20) If you answer at least one question correctly, I’ll give you an A.
(21) If you answer even one question correctly, I’ll give you an A.

• **Cf. Some vs. Any:**

Threat:
(22) If you make any/#some mistakes, I’ll break your legs.

Promise:
(23) If you answer any/some questions correctly, I’ll give you an A.

4. Crosslinguistic Evidence for the Relatedness of At Least and Even
Dutch, Japanese and Greek

- Dutch: zelfs maar = NPI ‘even’
  - lit. even only

- Japanese: -dake-demo = ‘at least’
  - lit. only even

- Greek esto is sometimes translated as ‘even’ and sometimes as ‘at least’ (Giannikidou 2007)

Dutch zelfs maar

- zelfs (= regular ‘even’)
  - top end of a scale
  - occurs in “positive” contexts

  (24) Hij heeft zelfs zeven vragen goed beantwoord.
  ‘He even answered seven questions correctly.’

- zelfs maar (= NPI ‘even’)
  - lower end of a scale
  - occurs in NPI licensing contexts

  (25) Ik denk niet dat hij zelfs maar een vraag goed beantwoord heeft
  ‘I don’t think he even answered one question correctly’

  (26) Heeft hij zelfs maar een vraag goed beantwoord?
  ‘Did he even answer one question correctly?’

  (27) Als je zelfs maar een vraag goed beantwoordt, slaag je.
  ‘If you answer even one question correctly, you’ll pass’

Japanese -dake-demo

- Imperatives
  (28) Aisu-dake-demo tabe-nasai.
  ‘Eat at least the ice cream.’

- Questions
  (29) Aisu-dake-demo tabe-ta? (positive bias)
  ‘Did you eat at least the ice cream?’

- Conditionals
  ‘If you answer at least one question correctly, you’ll pass.’
  Threat: # Iti-mon-dake-demo matigaeta-ra, asi-o oru-zo.
  ‘If you make at least one mistake, I’ll break your legs.’

Greek esto (from Giannikidou 2007)

- Imperatives
  (31) Fae esto to pagoto.
  ‘Eat at least the ice cream.’

- Questions
  (32) Efajes esto to pagoto?
  ‘Did you eat at least the ice cream?’

- Modals
  (33) Esto ke ena atomo bori na sikosi afo to trapezi.
  ‘Even one person can lift this table.’
5. Towards an Explanation of the Differences

• Does this explanation carry over to epistemic at least?

• Problem: epistemic at least doesn’t impose a preference ranking (at least not in simple sentences)

A union member can say:
(36) The manager fired at least two workers.

But still, (37) is odd as a threat (but judgements vary!):
(37) ?#If you fire at least two workers, we’ll go on strike.

Dutch minstens – only epistemic
(38) Als je minstens twee arbeiders ontslaat, gaan we staken.

Preference Ranking?

• Concessive at least
   Conventional implicature about preference ranking:
   \( \forall r, r' \in C \left[r > r' \iff r \text{ is preferred to } r' \right] \)

   (34) If you at least mow the lawn, I’ll give you your allowance.
   (35) #If you at least fail your math test, you’ll be grounded.

Positive Polarity?

• Positive bias has been observed for PPIs generally (Lakoff 1969)

   (39) If you make any / #some mistakes, I’ll break your legs.

• Both epistemic and concessive at least are PPIs

Clausemate negation:
   Epistemic: *He didn’t answer at least one question.
   Concessive: N/A

Matrix negation:
   Epistemic: *I don’t think he answered at least one question.
   Concessive: *I don’t think Mary at least won a silver medal.
• For epistemic at least, positive polarity follows from the epistemic uncertainty implicature:

$$\exists w[\text{Epist}(w,w') \land \exists q \in C[q > p \land q(w') = 1]]$$

“it is epistemically possible that some proposition q that ranks higher than p is true”

(40) * He didn’t answer at least one question.
  LF: not [at least [he answered one question]]

• Conventional implicature not affected by negation.
• So (40) conventionally implicates that it is epistemically possible that he answered more than one question.
• But (40) asserts that he didn’t answer any questions.
• Conflict between conventional implicature and assertion.

• But this explanation does not carry over to concessive at least.

• Maybe concessive at least is a speech act operator, and therefore resists embedding under logical operators like negation?

• But positive bias effect also occurs with concessive at least in questions and conditionals (where it IS embedded)

(41) Could you at least take out the garbage?
(42) I know you’re not going to clean the whole house, but if you at least take out the garbage, I’ll give you your allowance.

• Moreover, we don’t really know WHY PPIs cause the positive bias effect.

6. A Tentative (and Way Too Informal) Proposal

• Hypothesis: at least and NPI even differ in whether the target proposition p represents the lowest value on the scale or not.

  At least: $$\exists q \in C[q < p]$$
  Even: $$\neg \exists q \in C[q < p]$$
• With \textit{at least} the target proposition can’t be the bottom element of the scale.

• \textbf{Concessive at least}: This is part of the conventional implicature.
  
  iii. \(\exists q \in C \{ q < p \} \)
  
  “There is a proposition q that ranks lower than p”

• \textbf{Epistemic at least}: If the target proposition is at the bottom of the scale, the truth conditions make the statement uninformative.
  
  Truth conditions: \(\exists q \in C \{ q \geq p \land q(w) = 1 \} \)
  
  “there is a proposition q which ranks higher than or as high as the target proposition p and which is true”

• \textbf{NPI even} implicates that the target value is the \textit{lowest} of all the relevant possibilities.

\begin{enumerate}
  \item[(43)] Is Mary \textit{even} an assistant professor?
  
  assistant prof \(<\) associate prof \(<\) full prof

• \textbf{At least} implicates that the target value is \textbf{not the lowest} of all the relevant possibilities.

\begin{enumerate}
  \item[(44)] Is Mary \textit{at least} an assistant professor?
  
  postdoc \(<\) assistant prof \(<\) associate prof \(<\) full prof
\end{enumerate}

\begin{center}
\textbf{Kadmon & Landman 1993 on Any}
\end{center}

• \textbf{Any widens} the denotation of the noun.

• This widening has to result in a \textbf{strengthening} of the overall proposition.

• “Widening by strengthening” is only possible in a downward entailing context.

• This explains the NPI behaviour of \textit{any}.
Scales and Widening/Narrowing

- **NPI even** includes the lowest value of the scale, thereby causing a **widening** effect (just like any).
- **At least** excludes the lowest value on the scale, which results in a **narrowing** effect.

Assume that both **at least** and **even** must result in strengthening of the overall proposition.

- NPI even only occurs in **downward entailing** contexts.
- **at least** only occurs in **upward entailing** contexts.

Bias in Questions

(47) Is he **even** an assistant professor?
- By widening the scale, the speaker raises the chance of getting a positive answer (Krifka 1995: "lowering the threshold").
- This conversationally implicates that the speaker expects the answer to be negative.

(48) Is he **at least** an assistant professor?
- By narrowing the scale, the speaker lowers the chance of getting a positive answer ("raising the threshold").
- This conversationally implicates that the speaker expects the answer to be positive.

Krifka 1995 on Bias in Questions

- In **rhetorical questions**, the speaker tries to lower the threshold for a positive answer, showing that he is certain that the answer would be negative. For example, in *Did Mary ever lift a finger to help you?* the speaker wants to demonstrate how certain he is that Mary didn't help you at all by making the condition for a positive answer as weak as possible.

- In **information questions**, the speaker intends to construct the question in such a way that every suggested answer would be roughly yield the same amount of information increase. The principle can be illustrated by a game where one player draws a card from a deck of cards and the other has to guess it with as few questions as possible. It would be uneconomical to start with guesses like "Is it the seven of diamonds?"; it is better to start with questions like "Is it a seven?", or "Is it diamonds?". A question like *Have you ever been to China?* indicates that the speaker has a reason to prefer the more general question over any alternative, presumable because his information state is such that he expects a better overall information gain from an answer to the more general question.

(Promise vs. Threats)

- **Promise**: incentive to move **upward** on the scale.
  - (49) If you answer **at least** one question correctly, I'll give you a cookie.

- **Threat**: incentive to move **downward** on the scale.
  - (50) If you make **even** one mistake, I'll break your legs.
7. Conclusion and Further Questions

- Can this idea be formalized?

- Why the asymmetry between at least and even? (at least incompatible with threats, but even can be used in both promises and threats)

- What about regular even?

- How to deal with crosslinguistic data (e.g., zelfs maar vs. -dake-demo)? Is a compositional analysis of such particle combinations possible?

References


Büring, Daniel. 2008. The least at least can do. *WCCFL* 26, 114-120.


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