1. Two Kinds of *At least*

**Epistemic at least**

(Krifka 1999, Geurts and Nouwen 2007, Büring 2008)

1. Mary wrote *at least* four novels.
   *The speaker is uncertain about exactly how many novels Mary wrote*

2. Mary won *at least* a silver medal.
   *The speaker is uncertain about what medal Mary won*

**Another use of *at least*: Concessive at least**

3. Mary didn’t win a gold medal, but *at least* she won a silver medal.
   *(roughly) Although winning a silver medal is less preferable than winning a gold medal, a silver medal is satisfactory*

**At least is associated with a scale:**

A set of contrastive expressions of the same category

- **Non-entailment (Hirschberg 1985)**
  E.g. gold medal > silver medal > bronze medal

- **Entailment (Horn 1972)**
  E.g. n > … > 3 > 2 > 1
2. Distinguishing Properties

Syntactic diagnostic

(4) a. Mary won at least a silver medal.  
   b. Mary at least won a silver medal.  
   c. At least Mary won a silver medal.  
   d. Mary won a silver medal at least.

In the following,

- Epistemic: (4a) (prenominal at least)
- Concessive: (4c) (sentence-initial at least)

I   Compatibility with false higher scalar values

Epistemic: odd when higher values are known to be false
Concessive: no problem when higher values are known to be false

(5) Scale: gold medal > silver medal > bronze medal

E:  # Mary didn’t win a gold medal, 
    but she won at least a silver medal.

C:  Mary didn’t win a gold medal, 
    but at least she won a silver medal.

(6) Scale: n > … > 3 > 2 > 1

E:  # Mary doesn’t have three children, 
    but she has at least two.

C:  Mary doesn’t have three children, 
    but at least she has two.

II   Entailment with non-entailment scales

Epistemic: does not entail the truth of the target proposition
Concessive: entails the truth of the target proposition

(7) E: Mary is at least an associate professor.  
    → does not entail “Mary is an associate professor”  

C: At least Mary is an associate professor.  
    → entails “Mary is an associate professor”
III Preferability

Epistemic: no preference
Concessive: values higher on the scale are preferable

(8) \( E: \) Mary fired at least five employees.
\[ \rightarrow \text{no preference} \]

\( C: \) At least Mary fired five employees.
\[ \rightarrow \text{better to fire more employees} \]

IV “Settling for less”

Epistemic: neutral interpretation
Concessive: has a “settle for less” interpretation

(9) \( E: \) Phelps won at least eight gold medals.
\[ \rightarrow \text{neutral} \]

\( C: \) At least Phelps won eight gold medals.
\[ \rightarrow \text{winning eight gold medals falls short of an intended goal or standard} \]

V Scalar Implicature (SI)

Epistemic: the regular SI is not available
Concessive: the regular SI is available

(10) Mary wrote four novels.
\( \text{SI: } \) Mary didn’t write more than 4 novels

(11) \( E: \) Mary wrote at least four novels.
\( \text{No SI} \)

\( C: \) At least Mary wrote four novels.
\( \text{SI: } \) Mary didn’t write more than 4 novels

VI Lexical differentiation

- English
  (12) at least \( \rightarrow \) E or C (at the very least \( \rightarrow \) only E)

- Dutch
  (13) tenminste minstens, op z’n minst \( \rightarrow \) E or C only E

- Japanese
  (14) sukunaku-to-mo -dake-demo \( \rightarrow \) E or C only C
Summary

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3. The Semantics of *At Least*

Assumption 1

Kripka’s (1999) Compositional Projection Rules

E.g. gold medal > silver medal > bronze medal

Mary won a gold medal >
Mary won a silver medal >
Mary won a bronze medal

Assumption 2

*At least* is a sentential operator

(15) Surface: Mary won at least a silver medal.

LF: at least [Mary won a silver medal]

Scale:
Mary won a gold medal >
Mary won a silver medal >
Mary won a bronze medal

Epistemic *at least*

(Based on Kripka 1999, Geurts and Nouwen 2007, Büring 2008)

Truth conditions

\[ \exists q \in C [q \geq p \land q(w) = 1] \]

“there is a proposition q which ranks higher than or as high as the target proposition p, and which is true”

Conventional implicature

\[ \exists w'[\text{Epist}(w,w') \land \exists q \in C [q > p \land q(w') = 1]] \]

“it is epistemically possible that some proposition q that ranks higher than p is true”
(16) Surface: Mary won at least a silver medal.
LF: at least [Mary won a silver medal]
Scale: Mary won a gold medal > Mary won a silver medal > Mary won a bronze medal

Truth conditions
‘Mary won a silver medal’ or ‘Mary won a gold medal’ is true

Conventional implicature
The speaker considers it possible that ‘Mary won a gold medal’ is true

(17) Surface: At least Mary won a silver medal.
LF: at least [Mary won a silver medal]
Scale: Mary won a gold medal > Mary won a silver medal > Mary won a bronze medal

Truth conditions
p (= ‘M won a silver medal’) is true

Conventional implicatures
i. ‘M won a gold medal’ is preferred to p, etc.
ii. there is a proposition that ranks higher than p
iii. there is a proposition that ranks lower than p

Concessive at least

Truth conditions
p(w)=1
“the target proposition p is true”

Conventional implicatures
i. \( \forall r, r' \in C \ [r' > r] \) is preferred to r
“The scalar ranking reflects a preference ranking”
ii. \( \exists q \in C \ [q > p] \)
“There is a proposition q that ranks higher than p”
iii. \( \exists q \in C \ [q < p] \)
“There is a proposition q that ranks lower than p”
\( \rightarrow p \) is better than some other alternatives but not the best (i.e. “settle for less”)

(18) Surface: At least Mary fired five employees
LF: at least [Mary fired five employees]
Scale: Mary fired 6 employees > Mary fired 5 employees > Mary fired 4 employees > ...

Truth conditions
p (= ‘Mary fired five employees’) is true

Conventional implicatures
i. ‘Mary fired 6 employees’ is preferred to p, etc.
   = the more employees Mary fires, the better
ii. there is a proposition that ranks higher than p
iii. there is a proposition that ranks lower than p
Comparison of epistemic and concessive *at least*

I Compatibility with false higher scalar values

Epistemic: odd when higher values are known to be false
Concessive: no problem when higher values are known to be false

(19)  
**E:** Mary didn’t win a gold medal, but she won *at least* silver.  
**C:** Mary didn’t win a gold medal, but *at least* she won silver.

II Entailment with non-entailment scale

Epistemic: does not entail the truth of the target proposition  
Concessive: entails the truth of the target proposition

(21)  
**E:** Mary is *at least* an associate professor.  
→ does not entail “Mary is an associate professor”  
**C:** *At least* Mary is an associate professor.  
→ entails “Mary is an associate professor”

Conventional implicature of epistemic *at least*  
“it is epistemically possible that some proposition q that ranks higher than p is true”

Mary won gold  
Mary won silver  
Mary won bronze

Cf. (20)  
**E:** Mary didn’t win a gold medal, but she won *at least* bronze.

Concessive *at least* makes no reference to the truth value of higher ranked alternatives

Truth conditions of epistemic *at least*  
“there is a proposition q which ranks higher than or as high as the target proposition p and which is true”

→ ‘Mary is an associate professor’ may or may not be true

Truth conditions of concessive *at least*  
The target proposition is true  
→ ‘Mary is an associate professor’ must be true
III Preferability

Epistemic: no preference
Concessive: values higher on the scale are preferable

(22)  
\( E: \) Mary fired at least five employees.  
\( C: \) At least Mary fired five employees.

Conventional implicature of epistemic *at least*  
No reference to preference

Conventional implicature of concessive *at least*  
The scalar ranking reflects a preference ranking

IV “Settling for less”

Epistemic: neutral interpretation
Concessive: has a “settle for less” interpretation

(23)  
\( E: \) Phelps won at least eight gold medals.  
\( C: \) At least Phelps won eight gold medals.

Conventional implicatures of concessive *at least*  
The target proposition is better than some other alternatives but not the best (hence merely satisfactory)

\( \rightarrow \) inconsistent with the fact that ‘Phelps won 8 gold medals’ is a great achievement

Epistemic *at least* has no such implicature
V Scalar Implicature (SI)

Epistemic: the regular SI is not available
Concessive: the regular SI is available

(24) \( E: \text{Mary wrote at least four novels.} \)
    No SI

\( C: \text{At least Mary wrote four novels.} \)
\( SI: \text{Mary didn’t write more than 4 novels} \)

Epistemic at least asserts that either the target proposition \( p \) or a proposition ranked higher than \( p \) is true (Krifka 1999)

Concessive at least asserts that the target proposition \( p \) is true

Conversational implicature of \( p \)
\( \forall q \in C[q > p \rightarrow q(w)=0] \)
“every proposition \( q \) that ranks higher than \( p \) is false”

Prediction: (a) and (b) have the same assertion, thus same SI
(a) Mary wrote four novels.
(b) At least Mary wrote four novels.

Additional data

Neither epistemic nor concessive at least can be associated with an element that is at the bottom of the scale

(25) [Mary was in a swimming race. There were eight competitors including her.]

a. \( E: \#\text{Mary was at least eighth.} \)

b. \( C: \#\text{At least Mary was eighth.} \)

Truth conditions of epistemic at least
there is a proposition \( q \) which ranks higher than or as high as the target proposition \( p \), and which is true
\( \rightarrow (25a) \) is uninformative

Conventional implicature of concessive at least
“There is a proposition \( q \) that ranks lower than \( p \)”
\( \rightarrow \text{In (25b), ‘Mary was the eighth’ is at the bottom of the scale} \)
4. Summary

- Two types of \textit{at least}: epistemic & concessive
- Their distinguishing properties can be explained by their semantics
  - Epistemic: speaker’s epistemic uncertainty
  - Concessive: concessivity (“settle-for-less”) based on a preference ranking

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References